Chapter 13

LANGE'S ABSRACT RELATIVITY & INERTIAL REFERENCE FRAMES

During the period between 1885 and 1904, Ludwig Lange and others abandoned Newton's concept of 'absolute space.' They also abstractly and mathematically described and modified Galileo's sensory and empirical concept of relativity. The result was a very different abstract mathematical idealization of relative motion between two imaginary inertial reference frames (coordinate systems). In 1905, Einstein adopted this new mathematical model of relativity as the framework for his Special Theory of Relativity, which had absolutely nothing to do with Galileo's concepts of relativity.

A. Dissatisfaction with Absolute Space

What is meant by the word <u>space</u>? 'Space' has at least three separate and distinct meanings:

1. <u>Void</u>: That nothingness which is occupied by, and separates, material bodies.
2. <u>Place</u>: That certain part of a material body.
3. <u>Distance</u>: The physical interval between two material bodies or places.

With his concept of 'immovable absolute space,' Newton attempted to ascribe material properties to the infinite void of nothing that contains and separates material heavenly bodies. But how can 'nothing' be either moveable or immoveable? The term 'immoveable' implies that the void of celestial space remains in the same 'place.' But how can 'nothing' constitute or occupy a 'place,' or remain <u>there</u>? To paraphrase Gertrude Stein: "With empty space, there is no there, there." As stated by James Clerk Maxwell:

> "The arrangement of the parts of space can no more be altered than the order of the portions of time. To conceive them to move from their places is to conceive a <u>place to move away from itself</u>. But as there is nothing to distinguish one portion of time from another except the different events which occur in them, so there is nothing to <u>distinguish</u> one part of space from another except its relation to the

> place of material bodies. We cannot describe the time of an event except by reference to some other event, or the place of a body except by reference to some other body. All our knowledge, both of time and space, is essentially relative."[1] (Maxwell, 1877, p. 12)

Thus, again it is obvious that Newton's concept of 'immovable absolute space' was invalid and totally meaningless on its face. It follows from this conclusion that any concept or thing that depends upon 'immovable absolute space' for its existence, or measurement, is also invalid and meaningless. Among Newton's other concepts that depended upon 'immovable absolute space' for their existence or measurement were: absolute rest, absolute motion, absolute velocity, absolute direction of motion, absolute places, and 'relative spaces' in absolute space. (see Chapter 2)

Newton obviously confused the concept of void celestial space with the concept of material bodies, such as the Earth. A place is a position on a material body, but the position of a material body in celestial space is not a place. There is nothing that can be considered as an immovable reference body, place or system in celestial space itself. Thus, we cannot define 'relative spaces' nor measure from relative places in this infinite void. As Maxwell pointed out, we can only observe the relative positions of celestial bodies in the void of space and measure the relative change of positions (distances and motions) between them. [2]

On the other hand, a stake in the ground on Earth may be considered as relatively at rest or relatively immovable with respect to the Earth, so that the distances and positions of material objects on Earth can be measured from such stake and can be

[1] It should be noted that Maxwell made these comments 25 years before Poincaré, and 28 years before Einstein made similar comments.
[2] In this regard, Born also stated: "It is not space that is there and that impresses its form on things, but the things and their physical laws that determine space." (Born, p. 71) Likewise, "…German philosopher Gottfried Wilhelm Leibniz…argued that space is nothing but the relationship of the location of objects." (Rohrlich, p. 41)

defined as relative places, relative motions and relative directions of motion. (Chapter 2) Therefore, on Earth, some of Newton's spatial concepts of measurement can have meaning.

Newton attempted to justify his concept of an 'immovable absolute space' based on his concept of absolute motion,[3] and on his assumption that it was necessary as a stationary reference system for the description of his other laws and their time, place, and motion. For example, Newton believed that his first law of inertia, 'uniform rectilinear motion without applied force,' required a reference system absolutely at rest for its mathematical description. He failed to realize that all inertial bodies with uniform rectilinear motion empirically exemplify equivalent states of relative rest, and thus each is an appropriate reference frame for mathematically describing the others. (Jammer, 1954, pp. 97 – 98; 99 – 101)

With the gradual acceptance of Newton's laws of motion and mechanics, his companion concept of 'immovable absolute space' also became accepted as a fundamental law of physics during the 18th century. By the latter part of the 19th century it was even suggested by many scientists that the concept of stationary ether should be identified or equated with Newton's immovable absolute space. (Jammer, 1954, pp. 125, 141 – 142) Thus, the amorphous concept of immovable absolute space became materialized with the mythical substance of stationary ether.

During this same period of time, however, more and more scientists in England and France were reaching exactly the opposite conclusion: that "the concept of absolute

[3] Newton attempted to demonstrate his conjecture of absolute motion (and thus the validity of his concept of absolute space) with his famous rotating pail of water experiment, where an observer can infer the rotational motion of the pail without reference to any other body of reference, when he sees the water creeping up the sides of the pail. (Jammer, 1954, pp. 102 – 105) But Mach later argued against this proof. (*Id*. pp. 107, 139 – 141)

space was useless in physical practice.” (*Id*., p.138) This led to the paradoxical situation of theoretical adherence to the concept of absolute space on the one hand and its absence from practical physics on the other. (*Id*.)

B. Lange's Relative Inertial Frames

In 1885, German philosopher Ludwig Lange (1863 – 1936) suggested a hypothetical and mathematical way out of this paradoxical dilemma. Since it is impossible to determine a fixed spot in absolute space from which to measure, Lange proposed to eliminate Newton's 'immovable absolute space' as a conceptual foundation of physics. He replaced it with yet another fiction: a hypothetical system comprised of two inertial reference frames moving relative to one another within a limited space. (Born, p. 70) This fictional inertial reference system was invented strictly in the abstract, and without any reference to space or other bodies in the universe.

When two inertial reference frames (coordinate systems) were at rest together in Lange's abstract model, their common position and common motion was geometrically illustrated as two inertial systems, S and S', each with x, y, z coordinate axes located at their mutual zero coordinate position. (see Figure 13.1A) Algebraically, this scenario can be relatively described as $x = x'$, $y = y'$, $z = z'$, $t = t'$.

When the S' system theoretically <u>translates</u> at a uniform and rectilinear velocity away from the S system in the same (x) direction, their different positions and equivalent motions may be geometrically illustrated as shown on Figure 13.1B. [4] This second scenario may also be algebraically described, as follows:

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[4] This translatory motion when applied to Lange's relative inertial frames may also be referred to as an inertial 'boost motion.' In addition, Lange's concepts may be applied to 'displaced' or 'rotated' inertial motion. (see Figures 13.1C and 13.1D)

S system: $x = x' + vt$, $y = y'$, $z = z'$, $t = t'$, and

S' system: $x' = x – vt$, $y' = y$, $z' = z$, $t' = t$,

where x indicates the relative position of inertial system S theoretically at rest at the zero coordinate position; x' indicates the relative position of inertial system S' uniformly translating relative to x at the relative velocity of v in the same (x) direction; vt indicates the distance traveled by S' at velocity v relative to S during a time interval (t); and t = t' indicated the same absolute instant of time for the occurrence of events on either frame of reference, as theoretically perceived by all 19th century observers on each frame. (see Chapter 14)

Therefore, in place of a fixed spot in stationary absolute space, a mathematician substitutes the fiction that an inertial frame located at the origin point of a coordinate system in limited space is ‘stationary' relative to another inertial frame (coordinate system) that translates uniformly and rectilinearly away from it. Thence, from this fictional ‘stationary' inertial frame (like a stake in space), the mathematician can measure, compute and relate magnitudes of position and velocity between the two relatively moving inertial frames and within such limited space. These hypothetical coordinate frames became known as ‘inertial reference frames.' (Rohrlich, p. 43) By means of this idealized fiction, the mathematician can create an abstract mathematical certainty and absoluteness with respect to such measurements of position and velocity that does not exist in the real world.[5]

In Lange's view, by means of this fictional substitution, "the essential…content of

[5] For example, idealized relative velocities measured in Lange's abstract model never consider or experience the forces of gravity, the resistance of air and friction, temperatures, pressures, wind, weather, the myriad of other velocities of the Earth relative to other celestial objects, nor the direction of such relative velocities.

the law of inertia, and with it the whole of mechanics, retains its full physical meaning...An 'inertial system' is a coordinate system in respect to which Newton's law of inertia holds."[6] (Jammer, 1954, pp. 138 – 139) In effect, "in order to give [mathematical] meaning to the first law of motion we do not need absolute space and absolute motion but we can understand it entirely in terms of relative motion" (Rohrlich, p. 43), and limited space.

Thus one major reason why late 19th century mathematicians liked Lange's abstract model was because it gave them a mathematical way to theorize about celestial space and inertial motion in limited, specific and relative ways, rather than as an uncertain absolute place or absolute motion. (Rohrlich, p. 43) Their equations would not work with amorphous absolute space and absolute motions, any more than they work with the uncertain and intangible concepts of infinity or eternity.

Why were inertial frames (sometimes called 'Galilean frames') adopted as the standard frames of reference in Lange's model, rather than some other type of frame? Because if viewed from accelerating or arbitrarily moving bodies (frames), "the laws of mechanics would appear hopelessly complex…[and the] study of mechanics would present tremendous difficulties."[7] (D'Abro, 1927, p. 107) On the other hand, when referred to inertial or Galilean frames, "mechanical phenomena, including the planetary motions, were susceptible of being formulated by very simple [mathematical] laws."[8]

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[6] These, of course, are idealistic conclusions.

[7] Laws formulated from these non-inertial bodies would be compounded and complicated with accelerations, arbitrary herky-jerky motions, as well as the phenomena of centrifugal (outward) forces and Coriolis (sideways) forces. (D'Abro, 1927, p. 107)

[8] This was another reason why late 19th century scientists liked Lange's inertial reference frames: it was because of their simplicity and intuitiveness. There were no sensors or computers during these periods that could sense, sort out and compute the magnitudes of accelerating or arbitrary motions. Uniform rectilinear velocities allowed the accelerating motions and magnitudes of Newton's second law to be understood intuitively and expressed mathematically simply and elegantly.

(*Id*., p. 107) Thus, "it was essential to classical science that the fundamental law of mechanics [vis., Newton's second law of motion] should remain the same in all Galilean [inertial] frames…[so that] similar conclusions…[could] apply to all the mechanical laws."[9] (D'Abro, 1950, p. 113)

Not only did Lange's relativistic model provide mathematicians with relative inertial motions and with finite limited space intervals, it also provided them with relative instants in time and finite limited time intervals between inertial reference frames. Thus, it provided mathematicians with an idyllic framework for mathematically precise theoretical measurements between inertial frames.[10] "Lange's suggestion…was hailed by his contemporaries as an outstanding contribution to the foundation of physics." (Jammer, 1954, p. 139)

When Newton's second law of motion (force and acceleration) was theoretically applied in hypothetical experiments conducted upon each of Lange's two imaginary inertial reference frames, the resulting algebraic laws became known as 'Newtonian mechanics'…in order to distinguish them from the prior empirical laws of 'Newton's mechanics' which were theoretically dependent upon absolute space and absolute time. (Rohrlich, pp. 43 – 44) However, this so-called distinction was also a myth, because relative motion "was used in actuality all along by Newton as well as his successors since absolute space was never actually available as a reference frame."[11] (*Id*., p. 44) Thus, Lange's relativistic concept merely formalized the existing theoretical situation in

[9] Remember, we pointed out in Chapter 5 that Galileo's Relativity was not really a fundamental law of physics or motion, but rather only a convention of convenience. After 1885, the inertial motions of Galileo's Relativity (in the form of 'inertial frames') also became a mathematical necessity.
[10] However, late 19th century mathematicians did not realize the full physical and mathematical significance of these relative times for such measurements until Einstein explained them in 1905. (see Chapter 25)
[11] "People were simply not fully aware of this situation." (Rohrlich, p. 44)

abstract terms.[12]

It has been suggested that Lange's two abstract inertial coordinate frames simulate the ship in port on the inertially moving Earth, and the same ship later moving uniformly and rectilinearly away from the relatively stationary port in Galileo's Relativity. (Figure 5.3) "They are the frames which are not subject to acceleration, i.e. on which no forces are acting." (Rohrlich, p. 43) The scientific conclusion was that Lange merely reinterpreted Galileo's Relativity and inertia "in a relativistic way." (see Rohrlich, p. 43; Guilini, p. 14)

Did this historical connection with Galileo's Relativity mean that Lange's relativistic model and Galileo's Relativity were basically the same concept? Hardly. Lange's new abstract model was very different from the classical concept of Galileo's Relativity in many critical respects. First, in Lange's model there were two co-moving inertial reference frames rather than just one inertial body (the ship) in two equivalent states of uniform rectilinear motion. Second, in Lange's idealized model both reference frames were stipulated to be inertial, whereas in Galileo's Relativity the uniformly moving ship was non-inertial: its uniform velocity was caused by a force (the wind). Nor was the motion of the Earth really inertial, because of the forces of gravity impressed upon it.

Third, in Galileo's Relativity there was one observer in two different positions, whereas in Lange's model theoretically there were two different observers in two different positions (frames). Fourth, and very importantly, the one observer in Galileo's

[12] Therefore, the rationalization that Lange's relativistic model was invented "to rid Newton's mechanics of the need for absolute space and absolute time" had little merit. (Rohrlich, p. 43) This was especially true in light of the fact that 'absolute time' (the same instant of occurrence for a local and a distant event) remained in Newtonian mechanics and in the mathematical form of Galileo's Relativity as equations: $t' = t$ and $t = t'$.

Relativity describes two different events (accelerations) in two different locations, whereas the two observers in Lange's model each describe the same event as viewed from two different positions (perspectives). (see Figure 14.1) Fifth, the observer in Galileo's Relativity describes his same physical experiences at two different positions based on his empirical perspective and sensory illusion of rest, whereas the two observers in Lange's model mathematically measure and describe the positions of one event from two different frames (perspectives). These last three differences would later become critical for Einstein's Special Theory.

Sixth, in Lange's model one inertial system was assumed to be stationary (at rest) relative to the other. This was a mathematical fiction for purposes of algebraic description, measurement and computation. Whereas, in Galileo's Relativity, the only purpose of the same ship being stationary or at 'rest' in port and then uniformly moving away from the port was to empirically demonstrate that each illusion of rest was an equivalent state of motion that produced visually similar or covariant accelerations on it…so that the observer could not tell from observing such accelerations whether or not he was moving. On the other hand, the observer's ability to tell whether or not he was moving played no part in Lange's 1885 abstract model. Theoretically and mathematically it was irrelevant.[13]

Seventh, a primary purpose of Galileo's Relativity was to demonstrate the physical and empirical co-variance of different spatially separated accelerated motions and thereby to demonstrate the invariance of Newton's second law of motion on different

[13] The paradoxical 1887 M & M null result stimulated the attempt by scientists to discover the absolute motion (velocity) of the Earth relative to the stationary ether. Therefore, after 1887, a terrestrial observer's ability to detect or prove (theoretically or by experiment) that the Earth was moving relative to the ether at an absolute velocity became a cause celeb for theoreticians and mathematicians and their mathematical applications of Lange's reference frames, which strangely enough were irrelevant for this purpose.

bodies with equivalent sensory states of motion. With Lange's model this purpose was neither primary, nor required, nor even possible. The main purpose of Lange's model was that of mathematical measurement,[14] whereas measurement was irrelevant to Galileo's Relativity.

Eighth, the relative velocity of the ship at any position was irrelevant to Galileo's Relativity. The only requirement was that the motions of the ship were uniform and rectilinear. On the other hand, with Lange's model the relative velocity of the two inertial frames, v, was of primary importance because when v was combined with the time interval traveled, t, it produced the all important distance traveled, vt, which was necessary for mathematical measurement. Ninth, the distance traveled was irrelevant to Galileo's Relativity. The only thing that was relevant was that the uniformly moving ship was spatially separated from the port. On the other hand, the distance traveled by the moving frame in Lange's model was critical for purposes of measurement from one frame to the other.

Tenth, the time interval traveled and the instant in time on the ship at either position was irrelevant to Galileo's Relativity. Whereas, with Lange's mathematical model the time interval traveled was important for purposes of measurement, and the instant in time at each spatially separated position should have been important for measurement, as was pointed out by Einstein in 1905.[15] Lastly, Galileo's Relativity was

[14] One primary purpose of such mathematical measurement was to algebraically demonstrate that the same laws of mechanics were valid and invariant in different inertial reference frames. Einstein would later attempt to expand this mechanics concept to include electrodynamics (the velocity of light), and he then characterized this mathematical invariance of physics laws as mathematical 'covariance.' (see Einstein, *Relativity*, p. 48)

[15] There was another difference between Lange's Relativity and Galileo's Relativity. With Galileo's Relativity the inertial bodies did not have to be moving linearly relative to each other. Rather, such inertial bodies could even be moving perpendicularly relative to one another.

totally sensory and empirical in nature, whereas Lange's model was totally abstract and mathematical. Lange's idealized model was totally separated and isolated from the environment and empirical influences of reality, such as gravity, pressure, temperature, wind, climate, and friction. Without a sensory or empirical foundation it cannot independently be stated that Lange's model was based upon experience. Nor can it independently be stated that all inertial frames are equivalent states of motion, or that one inertial observer cannot tell whether he is moving or at rest. Nor can it independently be stated that no mechanical experiment performed entirely on one inertial reference frame cannot detect whether such frame is moving.[16] All of these conclusions depend entirely upon the classical and very different sensory and empirical concept of Galileo's Relativity.

For all of the above reasons, it becomes obvious that Lange's idyllic abstract model was designed to perform very different theoretical functions[17] and was a completely different concept than Galileo's sensory and empirical principle of relativity, which merely described reality for different specific and limited purposes.

C. Application of Lange's Inertial Reference Frames

Why did we just describe the many differences between Lange's abstract model and Galileo's Relativity? Because in 1905, Einstein adopted Lange's very different abstract (coordinate) version of relativity as the theoretical framework for his own mathematical Principle of Relativity. However, in the process Einstein (very

[16] This statement, incorrectly generalized to include EM experiments, became the foundation for Poincaré's and Einstein's principles of relativity. (see Chapter 21)

[17] In Lange's model, an inertial reference frame served at least three separate functions: it simulated a reference body with respect to which a distant observer might measure; it was the place where a local observer and his system of coordinates resided; and it was a uniformly moving platform upon which an accelerated event might occur, but it could not demonstrate the empirical covariance of Newton's second law. Galileo's Relativity only included the last function, and it did demonstrate such empirical covariance.

importantly) characterized his own Principle of Relativity as merely an extension of Galileo's Relativity, even though it applied (not only to mechanics, but) to all of physics as well…including the velocity *c* of light. (Einstein, 1905 [Dover, 1952, pp. 37 – 38]; Einstein, *Relativity*, pp. 15 – 18, 23) Einstein then further radically modified Lange's model of relativity by applying the Lorentz transformation equations to it. He then claimed that his Special Theory was just an attempt to reconcile the principle of relativity for mechanics with the constant velocity of light at *c*. All of these invalid rationalizations aside, Einstein was really attempting to reconcile his own radically different mathematical version of relativity with his own *ad hoc* concept for the absolute propagation velocity of light. (see Chapters 19, 20, 21 and 23) In the final analysis, Galileo's simple sensory and empirical concept of relativity had absolutely nothing to do with Einstein's abstract mathematical Special Theory. (Chapters 19 and 24)

By 1887, an algebraically modified version of Lange's relativistic model was being used by German mathematician Woldemar Voigt (1850 – 1919) for his paper on the Doppler theory. (Pais, p. 121) In 1892, Dutch mathematician H. A. Lorentz used Lange's abstract model for his relativistic contraction of matter theory. (Miller, pp. 25 – 27) In 1895, Lorentz used the classical equation $x' = x - vt$ as part of a set of radical transformation equations for his contraction of matter theory, but he then regarded such transformation equations "only as a convenient mathematical tool for proving a physical theorem…" (Pais, pp. 124 – 125) In 1898, Irish mathematician Joseph Larmor (1857 – 1942) used a radically modified algebraic version of Lange's model for a set of transformation equations in an attempt to confirm Lorentz's 1895 contraction theory. (*Id*., p. 126) In 1899, Lorentz used Lange's model to devise yet another radical set of transformation equations for his contraction theory, but for several years he did nothing with them. (*Id*., p. 125)

During 1902, French mathematician Henrí Poincaré (1854 – 1912) published a book, entitled *Science and Hypothesis*, wherein he talked about 'relative motion,' 'relative positions,' 'relative velocity,' 'the principle of relative motion,' 'the relativity of space,' and 'the principle of relativity.'[18] (Poincaré, 1902, pp. 90, 112 – 114, 243 – 244) Then, in 1904, at the St. Louis World's Fair, Poincaré suggested that the mechanics principle of relativity should be expanded so as to include all of physics (Lagunov, p. 25), he specifically defined what he meant by the 'principle of relativity' (see Chapters 16 and 21), and he ended a lecture with the statement: "Perhaps we must construct a new mechanics…" (Pais, p. 128) By May of 1904, after much prodding by Poincaré, Lorentz published a new version of his relativistic contraction of matter theory using Lange's abstract version of relativity as the theoretical framework and his own 1899 radical transformation equations as a mathematical form of proof. (see Chapter 16)

As previously explained, in June 1905, Einstein adopted for his own Special Theory of Relativity, the basic concepts of Lorentz's 1895 relativistic contraction of matter theory, Lange's abstract version of relativity as his framework, and Lorentz's radical 1904 transformation equations. By this time, June 1905, Galileo probably would not have recognized his own simple sensory and empirical concept of relativity.[19]

If it had not been for Lange's modified abstract model of Galileo's Relativity, Lorentz probably would not have been able to theoretically construct his relativistic

[18] However, in 1902, Poincaré did not specifically define such concepts.
[19] The modifications to, and transformation equations for, Galileo's Relativity (invented by Lange, Voigt, Larmor, Poincaré, Lorentz, Einstein and others) were completely new *ad hoc* mathematical concepts, with very different purposes than Galileo's Relativity, all without any direct empirical support. In the process, the original purposes for Galileo's simple sensory and empirical concept were totally abandoned.

contraction theory and his relativistic transformation equations in 1904. In this event, Einstein could never have adopted Lange's, Lorentz's and Poincaré's relativistic concepts, and probably never would have invented his own Special Theory of Relativity in 1905.

What is the current relevance of Lange's 1885 abstract model of Galileo's Relativity? Its sole relevance is that it was adopted by Einstein in 1905 as the abstract framework for the construction of his Special Theory. Other than this, Lange's abstract model of Galileo's 17th century relativity concept does not appear to retain much (if any) current or independent relevance.

Why is it necessary for us to learn and understand the exact criteria for, and the difference between, Galileo's Relativity and Lange's modified abstract model thereof in the 21st century? Because Einstein theories of Special Relativity and General Relativity evolved from them and were entwined with their concepts, and because Einstein's relativistic theories were predicated upon the applicability of Galileo's Relativity to electromagnetics, optics, electrodynamics and the velocity of light. Why was Galileo's empirical concept of relativity so important to Einstein? Because Einstein needed some empirical basis for his mathematical Special Theory so that it would not appear to be completely *ad hoc*.

If Einstein's relativistic theories were not based on Galileo's empirical concept of relativity, and if Galileo's Relativity has no relevance to electromagnetics, optics, electrodynamics and the velocity of light (which it does not), then where does this leave Einstein's relativistic theories? The answer is: completely without any empirical foundation whatsoever. Einstein's relativistic theories are therefore totally *ad hoc*. [20]

At this point it is recommended that the reader turn to Chart 24.1 in order to study and understand the various different concepts and definitions of 'Relativity.' We shall repeatedly refer to all of these concepts in great detail in later chapters.

[20] When critically analyzed and scrutinized in later chapters, Einstein's Special Theory appears to be nothing more than an arbitrarily constructed mathematical exercise, with no application whatsoever to reality.

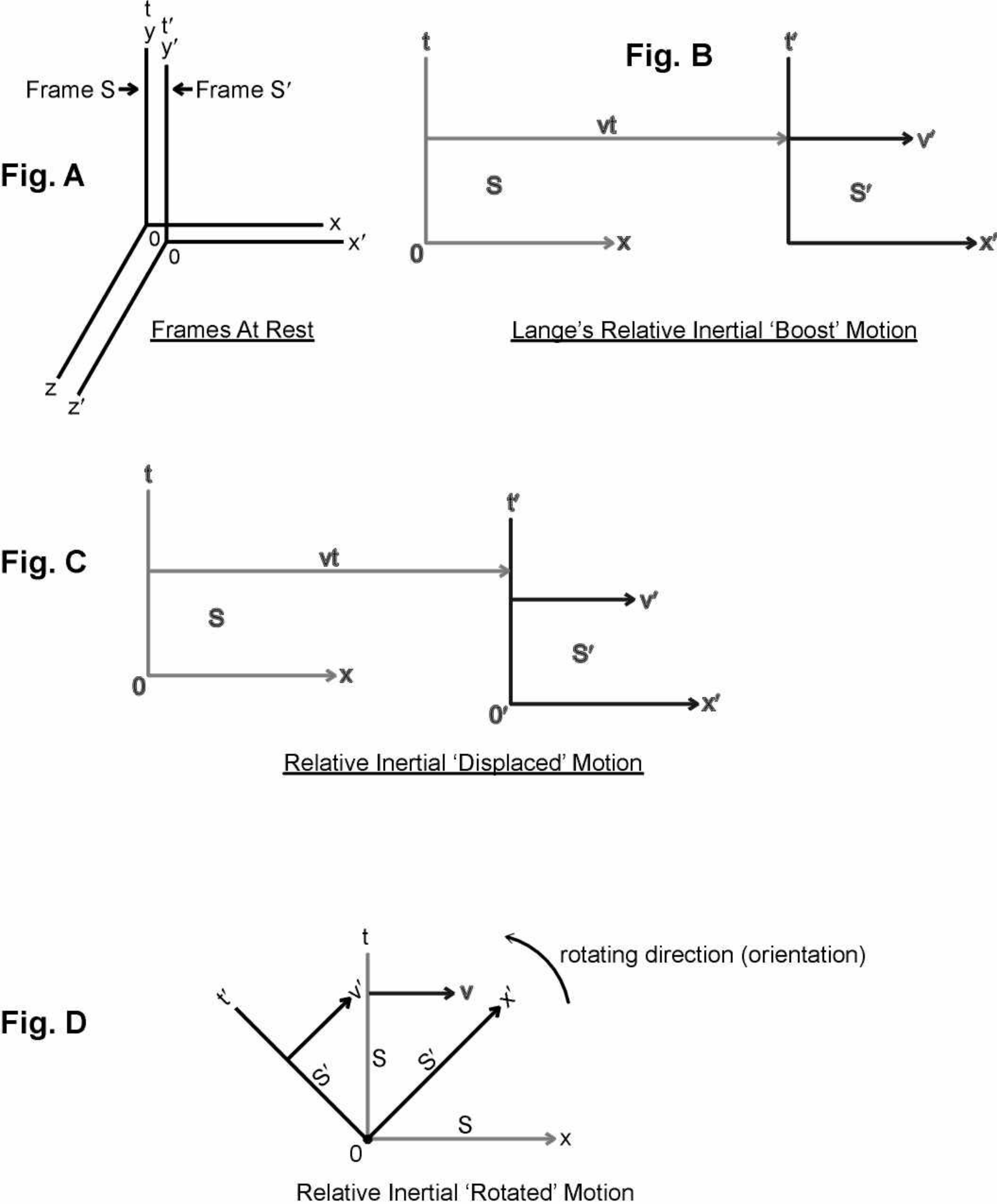

Figure 13.1 The Family Of Relative Inertial Frames