Chapter 29

THE RELATIVISTIC COMPOSITION OF VELOCITIES

From the Lorentz transformation equations, Einstein derived a relativistic formula for the kinematic addition of two velocities in the same direction, which algebraically never could exceed the velocity c. Einstein claimed that this formula mathematically confirmed his second postulate concerning the constant propagation velocity of light at c in every inertial frame. Later he claimed that the 1851 Experiment of Fizeau empirically confirmed the validity of this formula. But it turns out that neither claim is correct.

A. Einstein's Relativistic Formulas for Composition of Velocities

In Section 5 of his Special Theory, entitled "The Composition of Velocities," Einstein claimed to have derived from his two fundamental postulates, with the help of his Lorentz transformation equations, a new addition of velocities law for the transformation if velocity components in inertial systems S and S'.[1] In his 1905 Special Theory, Einstein only considered the relativistic transformation of velocities parallel to the direction of relative motion v of S and S' in the x – x' direction.[2] (see Resnick, 1968, p. 82)

First, Einstein asked the question: What is the velocity of a point moving at velocity w in S' as measured by S, where S' is moving away from S at the uniform relative velocity of v? His answer was, as follows:

> "It is worthy of remark that *v* and *w* enter into the expression for the resultant velocity in a symmetrical manner. If *w* also has the direction of the axis of X, we get
>
> $$V = \frac{v + w}{1 + vw/c^2}."$$

(Einstein, 1905d [Dover, 1952, p. 50]),

[1] In fact, it is quite obvious that Einstein did exactly the reverse. He realized that a 'relativistic addition of velocities transformation' formula could be constructed from Lorentz's transformation equations, and thereafter he used this formula to construct and justify his two postulates.

[2] For Einstein's transformations of velocities which are perpendicular or transverse to the direction of relative motion of S and S', see Chapter 32.

In this formula, V is the ‘relativistic addition of velocities’ for two bodies moving in the same parallel direction, as measured by S. “However, if the relative velocity v is not along the common direction of the x-axis, then Einstein’s symmetry [reciprocity] is lost.” (Miller, p. 260)

Based on this new relativistic transformation equation of velocities, Einstein concluded that: “from a composition of two velocities which are less than *c*, there always results a velocity less than *c*.” (Einstein, 1905d [Dover, 1952, p. 51]) Based on this mathematical result, Einstein had previously asserted in § 4 of his Special Theory: “that the velocity of light in our theory plays the part, <u>physically</u>, of an infinitely great velocity.” (*Id*., p. 48)

What was the empirical foundation for these mathematical conclusions? In 1905, there was probably only one type of experimental results that could be claimed to form an empirical basis. That was the highly speculative experimental and theoretical work with ‘electromagnetic mass’ by Kaufmann, Abraham, Lorentz and others, where it mathematically appeared that *c* it might be the ultimate speed limit for electromagnetic mass. (see Chapter 18) In Chapter 31, we will empirically demonstrate that the velocity of light at *c* may indeed be the limiting velocity for a material particle or body, but not because of the Lorentz transformation equations nor a body’s relative kinematic velocity.

Because the first part of Einstein’s second postulate, vis. that all inertial observers will measure the same velocity *c* for light propagating through their reference frames, is <u>embedded</u> in his new relativistic formula for the addition of velocities (see Miller, p. 261), Einstein then concluded:

“It follows, further, that the velocity of light *c* cannot be altered by composition with a velocity less than that of light. For this case we obtain

$$V = \frac{v + w}{1 + vw/c} = c."$$

(Einstein, 1905d [Dover, 1952, p. 51])

Here, Einstein is also claiming that his transformation formula for the composition of velocities is mathematical confirmation for his second postulate concerning the invariant propagation velocity of light at *c* in all inertial frames.

This result is not at all amazing. In fact, it should mathematically be expected. (Miller, p. 261) As Resnick also pointed out: "We know that an assumption [postulate] used to derive the transformation formulas was exactly this result: that is, that all observers measure the same speed *c* for light." (Resnick, 1968, p. 80) In other words, the first part of Einstein's second postulate mathematically results in itself, in the algebraic form of Einstein's relativistic formula for the addition of velocities. Again, so much for the mysteries and manipulations of mathematics.[3]

Was there any empirical foundation for Einstein's second conclusion: "that the velocity of light *c* cannot be altered by composition with a velocity less than that of *c*"? In his book, *Relativity,* Einstein asserted that Michelson's null results were empirical confirmation that the velocity of light at *c* in Michelson's apparatus when added to the solar orbital velocity of the Earth at 30 km/s does not change the velocity of light at *c* in the direction of such motion.[4] The empirical reason for such null results that Einstein described as the "right one" was the contraction of Michelson's apparatus in the direction

[3] In § 4 of his Special Theory, Einstein asserted that: "For velocities greater than that of light our deliberations become meaningless…" (Einstein, 1905d [Dover, 1952, p. 48])

[4] Many of Einstein's followers also cite Michelson's null results as experimental confirmation of Special Relativity. (see Resnick, 1968, p. 37)

of such motion.[5] (Einstein, *Relativity*, pp. 58 – 60) Since we now know the real reasons for Michelson's null results (Chapters 9 – 12), we should also realize that Michelson's null results were not empirical confirmation for Einstein's relativistic composition of velocities formula.

B. Is the 1851 Experiment of Fizeau Confirmation of Einstein's Formula?

In 1907, Max von Laue deduced Fresnel's ether drag coefficient from Einstein's Special Theory, which he considered "as a further illustration of Einstein's addition theorem for velocities." (Miller, p. 263; Rindler, p. 69) Based on Laue's deduction and conclusion, Einstein devoted the entire Chapter 13 of his 1916 book *Relativity* to Fizeau's 1851 'experimental verification' of Fresnel's mathematical coefficient. Such chapter was entitled: "Theorem of the Addition of the Velocities. Experiment of Fizeau." (Einstein, *Relativity*, pp. 43 – 46) Einstein also asserted that the comparison of his formula for the relativistic addition of velocities with the results of Fizeau's 1851 experiment resulted in an elegant confirmation and a "crucial test in favor of the theory of relativity." (*Id*., p. 46) Many of Einstein's followers also cite Fizeau's 1851 experiment (and repetitions thereof) as experimental confirmation of Special Relativity, and of his relativistic formula for the composition of velocities. (see Resnick, 1968, p. 37; Zhang, pp. 207 - 214)

Einstein began Chapter 13 of *Relativity* by conjecturing that, although his concepts of 'length contraction' and 'time dilation' only manifest themselves at velocities near the velocity of light, he would now easily derive from his 'relativistic addition of velocities' a quantity which could be spectacularly demonstrated and "elegantly

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[5] Einstein and his followers attempted to claim that such contraction was not physical, but rather only a result of the relativistic way that relative motion is measured. (see Chapter 28) But how can a mere illusion of measurement result in the physical contraction necessary for the reciprocal decreased time interval for the light to propagate at *c*? It cannot.

confirmed by experiment" at low velocities. (Einstein, *Relativity*, p. 43; also see Miller, p. 263) Einstein was, of course, referring to Fizeau's 1851 experimental confirmation of Fresnel's ether drag coefficient. He was also referring to Pieter Zeeman's 1914 and 1915 repetitions of the Fizeau experiment, which he claimed exactly agreed with his formula for the relativistic composition of velocities.[6] (see Einstein, *Relativity*, p. 46; Zhang, pp. 211 – 212, 281) Let us now scrutinize Fizeau's experiment and Einstein's Chapter 13, to see if Einstein's above predictions and assertions are confirmed by reality.

Einstein analyzed the 1851 Fizeau experiment as if it was a study in Galilean Relativity:

> "The tube plays the part of the railway embankment,…the liquid plays the part of the carriage,…and finally, the light plays the part of the man walking along the carriage…" (Einstein, *Relativity*, p. 45)

See Chapter 8 and Figure 8.3 of this book for a detailed discussion and illustration of the 1851 Fizeau experiment.

Einstein assumed that "light travels in a motionless liquid with a particular velocity w." (*Id.*) This empirical velocity w is, of course, a known quantity, and is used to determine the Index of Refraction. (see Figure 7.7) The velocity of the liquid in Fizeau's tubes (v) was also known to be approximately 7 meters/second. Einstein then asked what was the velocity W of the light in the liquid with respect to the tube T, "when the liquid…is flowing through the tube with a velocity v"? (*Id.*) Fizeau empirically observed in 1851 that the velocity W was greater (by 44% of the velocity of the liquid) in the direction of the flow, and that velocity W was less (by 44% of the velocity of the liquid) against the direction of the flow. (Gamow, 1961, p. 164)

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[6] The reason for this agreement was most likely because Zeeman used Special Relativity in arriving at his results.

In order to answer his own question as to the magnitude of velocity W, Einstein deduced the 'classical addition of velocities' from the Galilean transformations and obtained, W = v + w. This was the same formula that he used with his prior example of the man walking in the same direction as the carriage moving along a stationary embankment. (Chapter 19)

The ultimate question which Einstein wanted answered was which theorem for the addition of velocities better describes Fizeau's empirical results: A. The classical theorem for the addition of velocities (W = v + w); or B. His new relativistic formula for the addition of velocities,

$$W = \frac{v + w}{1 + vw/c^2} .$$

(*Id*., pp. 44 – 45, 46) Einstein then concluded that his new relativistic formula was the winner for purposes of description, and that "the agreement is, indeed, very exact."[7] (*Id*., p. 46) Let us now continue to examine and scrutinize these and other claims by Einstein.

There were three basic questions posed by the mysterious empirical results of Fizeau's Experiment: 1) Why is only a fraction of the velocity of the moving medium (water) transferred to the velocity of light propagating through it? Unlike light in a moving medium, the propagation of sound waves is directly proportional to the velocity of a wind. (Gamow, 1961, p. 162) 2) Why is such fraction related to the square of the Index of Refraction? 3) Which equation for the addition of velocities best describes the empirical results of the Fizeau experiment? There are numerous theories that attempt to answer the first two questions. (For example, see Pavlovic, Sections 13, 14, 19 & 20, and the author's suggestions in Chapter 8) Einstein had no answer for such questions.

[7] But Einstein never explained why Fizeau's paradoxical result occurs. On the other hand, the author's empirical quantum explanation of 'why,' is set forth in Chapter 8.

The third question is the only one that is really relevant to this Chapter. In *Relativity*, Einstein claimed that Fresnel's empirical equation $v' = v(1 - 1/n^2)$, which mathematically describes the empirical result of Fizeau's experiment, is equally well described by his relativistic theorem for the addition of velocities. (Einstein, *Relativity*, pp. 43-46) In footnote 1 on p. 46 of *Relativity*, he even suggested an algebraic manipulation that would make the approximation exact:

> "Fizeau found $W = w + v(1 - 1/n^2)$, where $n = c/w$ is the index of refraction of the liquid. On the other hand, owing to the smallness of vw/c^2 as compared with 1, we can replace (*B*) in the first place by $W = w + v(1 - vw/c^2)$, or to the same order of approximation by $w + v(1 - 1/n^2)$, which agrees with Fizeau's result." (*Id*., p. 46)

Why does Einstein's relativistic formula for the composition of velocities <u>appear</u> to so closely describe the empirical result obtained by Fizeau and described by Fresnel's coefficient? The answer is several fold: First, both formulae include the same values, vis., the velocity of light (c) *in vacuo*, the speed of light (w) in motionless water, and the velocity (v) of the material substance (water) relative to the tube. Secondly, the Index of Refraction, when plotted on a graph for all hypothetical velocities of light through different material mediums, <u>coincidentally</u> results in substantially the same graphic configuration as the Lorentz transformation, $1/\sqrt{1 - v^2/c^2}$. (compare Figure 29.1 with Figure 17.2) The two formulae also hypothetically result in substantially similar quantities. (compare Chart 29.2 with Chart 17.3)

Thirdly, and the most important reason why Einstein's relativistic formula for the composition of velocities <u>appears</u> to be so close to Fresnel's coefficient is the <u>low magnitude of v</u> (the velocity of the water in Fizeau's experiment) in both equations. Einstein asserted that "Fizeau found $W = w + v(1-1/n^2)$, where $n = c/w$ is the index of refraction of the liquid." (Einstein, *Relativity*, p. 46, F.N. 1) The approximate values in the Fizeau experiment were: w, the speed of light in the motionless water (226,000 km/s); *c*, the speed of light in a vacuum (300,000 km/s); v, the velocity of the water through Fizeau's tube (7 m/s); and W, the velocity of light propagating relative to the tube. (*Id.*, p. 45)

When we apply the above values to Fizeau's experiment, and to the equation which Fizeau found ($W = w + v(1 - 1/n^2)$), this results in the following magnitude for W: W = 226,000.003027422222 km/s. When we apply the same values of the Fizeau experiment to Einstein's relativistic formula for the addition of velocities ($W = v + w/1 + vw/c^2$) we get the following magnitude for W: W = 226,000.003027422169 km/s. Very close. However, when we apply the same values of the Fizeau experiment to the classical addition of velocities (W = v + w), we get the following magnitude for W: W = 226,000.007 km/s. Not so close.

Thus, the answer to Einstein's ultimate question concerning W is that his relativistic formula does better describe the result of the Fizeau experiment and the equation which Fizeau found than does the classical addition of velocities equation. From the above comparison, Einstein asserted the "conclusiveness of the [Fizeau] experiment as a crucial test in favor of the theory of relativity…"[8] (*Id.*, p. 46) But the magnitudes for Fizeau's equation and Einstein's formula are <u>not exactly</u> the same…Einstein's magnitude for W is <u>slightly less</u> than Fizeau's at the velocity of 7 meters/second for the water.

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[8] Many scientists agreed with Einstein: "that the mysterious empirical formula [of Fresnel and Fizeau] is a <u>direct result</u> of the theory of relativity." (see Gamow, 1961, p. 164)

Let us now see if the closeness of the magnitudes between Einstein's formula and Fizeau's equation holds for all possible velocities of the water. If we substitute the velocity of 1% of *c* for the velocity of the water (instead of 7 m/s), we get:

Einstein's formula	W = 227,287.7655 km/s
Fizeau's equation	W = 227,297.4666 km/s

At this increased velocity of the water, the magnitude of W in Fizeau's equation is somewhat more than in Einstein's formula.

On the other hand, if we substitute the velocity of 50% of *c* for the velocity of the water, we get:

Einstein's formula	W = 273,123.4866 km/s
Fizeau's equation	W = 290,873.3333 km/s

(see Chart 29.3) At this greatly increased velocity of the water, the magnitude of W in Fizeau's equation is significantly more than in Einstein's formula.

At the velocity of 1% of *c* for the water, Einstein's relativistic formula produces a velocity W, which is approximately 9.7 km/s less than Fizeau's equation. However, at the velocity of 50% of *c* for the water, Einstein's relativistic formula produces a velocity W, which is approximately 17,750 km/s less than Fizeau's equation. (Figure 29.4) At the velocity of 50% of *c* for the water, the close correlation completely disappears. Then, at the velocity of 99% of *c* for the water, the difference between the two equations becomes tremendous. (Figure 29.4)

Thus, the apparent closeness of the result between Einstein's formula and Fizeau's equation only holds for velocities of the water, which are <u>very low</u> relative to *c*. Contrary to Einstein's assertion, there is <u>nothing conclusive</u> about "the [Fizeau] experiment as a crucial test in favor of the theory of relativity." (Einstein, *Relativity*, p. 46) The 1851 Experiment of Fizeau is in no way an experimental confirmation for the validity of Special Relativity, the Lorentz transformations or Einstein's relativistic composition of velocities. (also see Pavlovic, Sections 13, 14, 19 & 20, with regard to similar conclusions)

In fact, the Experiment of Fizeau theoretically demonstrates just the opposite. When Einstein fudged his equation for the relativistic composition of velocities so as to make such velocities <u>agree exactly</u> with Fizeau's equation (Einstein, *Relativity*, p. 46, F.N. 1), he was in effect asserting the <u>identity</u> of his equation with Fizeau's equation. But as we see on Chart 29.3 and Figure 29.4, Fizeau's equation, when applied to very high velocities of the water, produces velocities for light in the medium of moving water well in excess of *c*. How can this apparent contradiction with Einstein's relativistic formula for the addition of velocities and with Einstein's kinematic conclusion for the limiting velocity of matter at *c* be reconciled or justified?[9]

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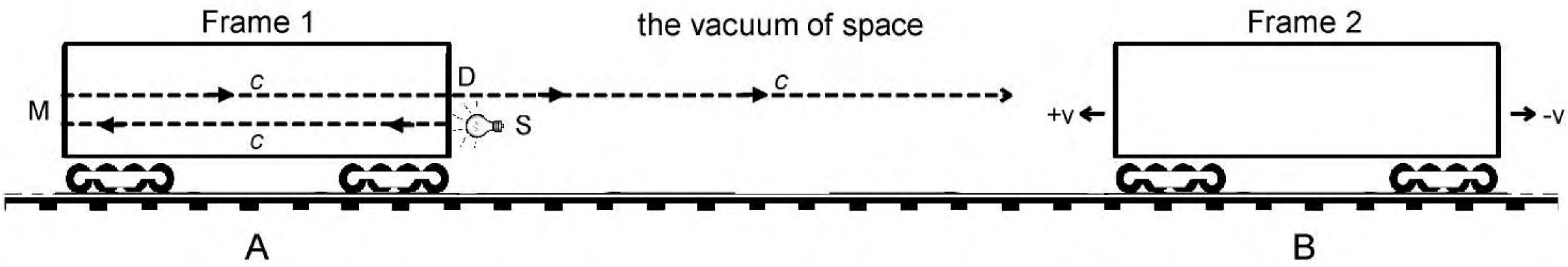
[9] If instead of moving water we refer to high energy moving particles, there the identity of Einstein's formula with Fizeau's formula can have some theoretical meaning even in the twenty-first century.

Figure 29.1 The Real Answer To Einstein's 1905 Paradox



The light ray is emitted from the source S at one end of the stationary Frame 1. It propagates over a finite distance d from S to the reflecting mirror M and back over the same finite distance d to the detector D in time t. 2d/t = *c* (300,000 km/s), the transmission velocity of light in a vacuum.

The same light ray continues propagating at its transmission velocity of *c* relative to its medium of the vacuum of empty space, toward Frame 2. If Frame 2 is moving at v toward the tip of the light ray, the light ray's relative velocity of propagation with respect to Frame 2 over such changing distance and time interval is *c* + v. On the other hand, if Frame 2 is moving at v away from the tip of the light ray, the light ray's relative velocity of propagation with respect to Frame 2 over such changing distance and time interval is *c* - v.

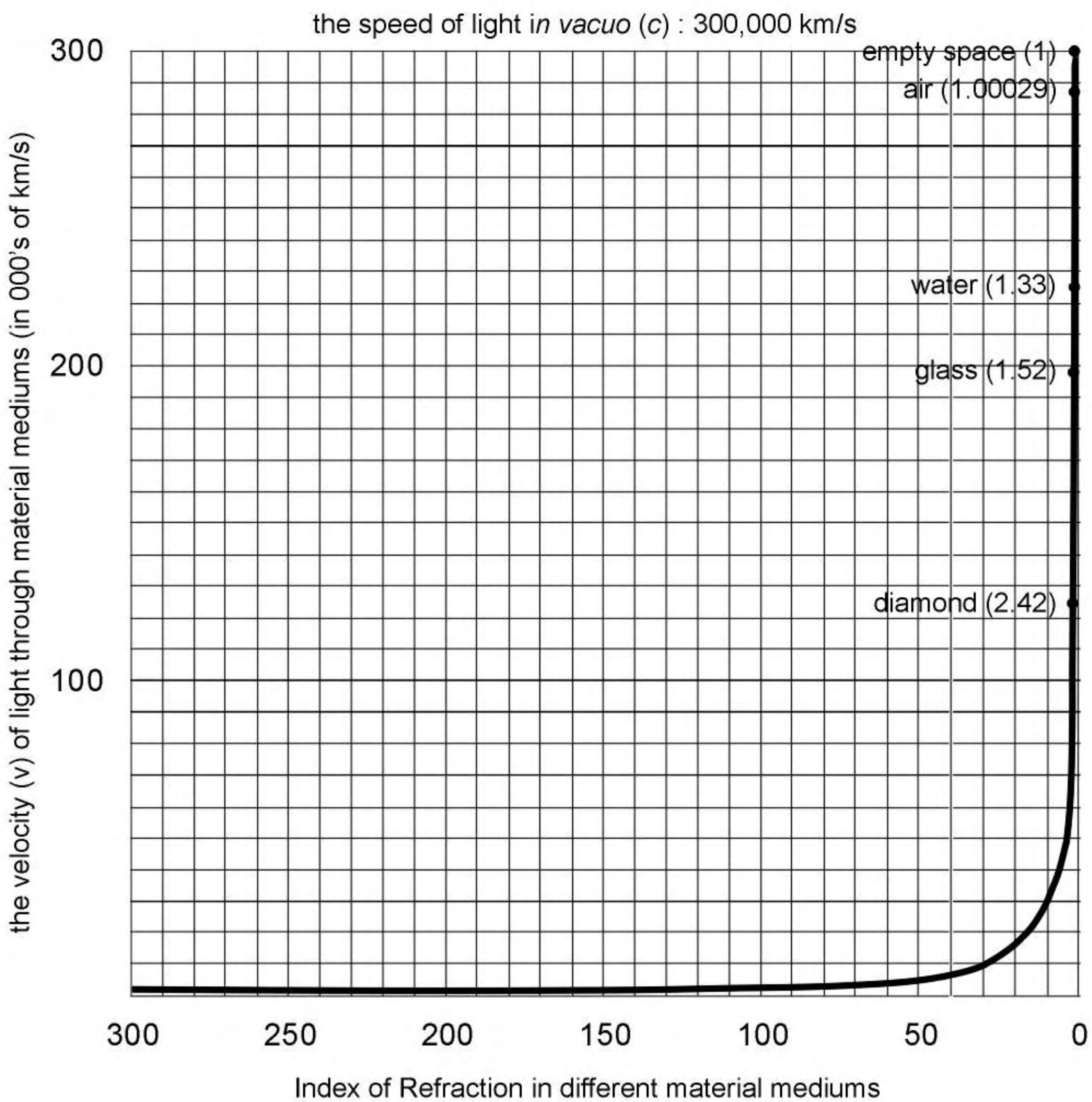


Figure 29.2 Hypothetical Graph Of The Index Of Refraction (n = *c*/v) For All Possible Material Mediums

Chart 29.3 The Velocity Of Light Through All Material Mediums, As Hypothetically Determined By The Index Of Refraction: n = *c*/v

Index of Refraction n = *c*/v	Velocity of light (v) through the medium (in 000's of km/s)	Medium
300	1	
150	2	a neutron star?
100	3	a black hole?
30	10	lead?
10	30	steel?
6	50	
3	100	
2.41935483	124	diamond
2.0	150	
1.52284263	197	glass
1.5	200	
1.32743362	226	water
1.09090901	275	
1.034482	290	
1.0169491	295	
1.003344	299	
1.0003334	299.9	air
1	300	empty space

Chart 29.4 Values for Fizeau's Equation As Compared To Einstein's Equation

		Velocity (W) of light in the medium of moving water	
		Fizeau's Equation	**Einstein's Equation**
		$\mathbf{W} = w + v\,(1 - 1/n^2)$	$\mathbf{W} = \frac{w + v}{1 + vw/c^2}$
Velocity (v) of the medium (water)			
as a % of *c*	km/sec	km/sec	km/sec
0%	0	226,000	226,000
1%	3,000	227,297.4666	227,287.7654
10%	30,000	238,974.6666	238,509.3167
20%	60,000	251,949.3333	248,551.5643
30%	90,000	264,924.0000	257,748.7765
40%	120,000	277,898.6666	265,881.1475
50%	150,000	290,873.3333	273,123.4866
60%	180,000	303,848.0000	279,614.3251
70%	210,000	316,822.6666	285,464.8625
80%	240,000	329,797.3333	290,765.3910
90%	270,000	342,772.0000	295,589.9880
95%	285,000	349,259.3333	297,843.4039
99%	297,000	354,449.2000	299,576.1255
99.99	299,970	355,733.6920	299,995.7792
100%	300,000	355,746.6666	

Figure 29.5 The Velocity Of Light In The Medium Of Water, Relative To The Velocity Of The Water