Chapter 3

TECHNIQUES & CONVENTIONS OF MATHEMATICAL MEASUREMENTS

During the 17th century, the positions, relative distances, and motions of bodies in space were defined and illustrated by Cartesian coordinates. When theoretically attached to the eyes of an observer, these mathematical coordinate systems (plus a hypothetical clock) were called 'frames of reference.' Observers at different positions have different perspectives of the relationships between material objects. The attempt to mathematically measure and relate different 'perceptions' of the same event was sometimes called 'Relativity.'

In this chapter we shall discuss some of the major classical techniques and conventions that were invented over the centuries to mathematically describe and measure physical phenomena. Many are still applied and referred to by theoretical physicists and pure mathematicians, as we shall discover throughout this treatise.

A. Positions, Coordinates, Frames & Motions

Ancient Egyptian land surveyors first developed the practical rules of space measurement and geometry by trial and error. Then, about 300 BCE, Greek scholar Euclid formalized the empirical concepts of geometry (based primarily on points, straight lines, triangles, and circles) into a system of mathematical axioms in order to measure land and construct buildings. It became known as "Euclidean Geometry." [1] (Goldberg, pp. 7-9) Another reason for such measurements was to determine the position of a material object or a physical event (i.e. a battle) in order to locate and describe it with precision.[2]

[1] Euclid's treatise on mathematics was entitled: the "*Elements*." The geometry that it described "originated from observational data and practical experience." (Logunov, p. 5) "The widespread use of Euclidean geometry was a measure of the truth of Euclidean axioms." (Goldberg, p. 68)

[2] The Earth was first measured (a process called "geodesy" by the early Greeks) depending upon the number of a man's paces or by the length of a chain or rod. Later it was measured by exact units (meters and yards). Time was originally measured by night and day, the phases of the moon, the seasons, and sundials. Later it was measured by exact units (seconds and hours). (Born, pp. 5 – 6)

The term ‘position’ may be defined as a specific material point in space at a specific instant in time. The concept of ‘position’ implies the measurement of distance from a tangible body of reference (such as a post on the surface of the Earth) to a material object or physical event. (see Figure 2.1A) “To establish the distance between two points on a rigid body [i.e. Earth]…is the basis of all measurement of length.” (Einstein, *Relativity*, p. 6) With regard to the measurement of material objects, Newton assumed that the concept of length or distance had a definite physical meaning, vis. that a yardstick had the same physical length for all observers regardless of their relative positions, visual perspectives or states of motion.[3] (Einstein, Princeton, 1922, pp. 25, 26) “Every description of events in space involves the use of a rigid body to which such events have to be referred.”[4] (Einstein, *Relativity*, p. 9) Such rigid body is often called a ‘body of reference.’

Newton faced the theoretical problem of describing positions and motions of material objects in empty space where there was no stationary body or place of reference. How could he physically or mathematically describe the position of a material body, its uniform inertial motion, its accelerated motions, its velocity or its trajectory, without some fixed place or point of reference? Newton’s solution of expediency was to merely postulate an immovable place of reference, ‘absolute space,’ which theoretically was

[3] In other words, for Newton the physical length and other physical dimensions of a rigid object were invariant properties of that object. The author asserts that this is a correct concept. On the other hand, Einstein (in 1905) challenged this assumption and conjectured (as a cornerstone for his Special Theory) that relative linear motion changes the physical linear distances on one moving object when viewed by an observer on another object moving linearly at a different velocity. (Einstein, 1905 [Dover, 1952, pp. 41 – 42]) However, as we shall discover in Chapters 26 and 28, Einstein’s ‘Relativity of Distance’ concept was only an illusion based on Einstein’s dubious techniques of measurement.

[4] In Chapter 22 we will discuss why this statement by Einstein has no real meaning with regard to the point of emission of a ray of light in space, and its velocity of transmission at *c* relative to such point of emission.

absolutely at rest.[5] (Chapter 2) Unfortunately, this artificial solution caused many theoretical problems for physics during the next three centuries.[6]

During the early 17th century, French philosopher René Descartes (1596-1650) devised a simple system to abstractly describe and illustrate on a piece of paper the position of a point or an event in space. He used a grid of rectilinear intersecting lines. Each line on the grid was assigned a different number, and the point where any two lines intersected was called a 'coordinate.'[7] (see Figure 3.1A) The origin or zero point on the grid played the part of a body of reference. It was the point of reference to which all coordinates referred. (Goldberg, p. 73)

Descartes marked the position of a material object or physical event on this grid by determining its three spatial distances or dimensions (width, length, and height) from the zero point of reference. These dimensions are represented by three perpendicular spatial coordinate axes: x, y, and z.[8] He depicted the distance between points (positions) by other lines. (Harrison, p. 126) This abstract graphical method of location and measurement became known as 'Cartesian coordinates.'[9]

Cartesian coordinates can be used to measure two-dimensional distances between two points on a plane (i.e. on a football field), usually by means of the Pythagorean Theorem. (Figure 3.1A) Likewise, Cartesian coordinates can be used to measure three-

[5] Newton also referred to absolute space as 'mathematical space.'
[6] For example, D'Abro concluded that this invalid postulate by Newton "rendered inevitable a belief in some real absolute medium, space or the ether, from which rotation and acceleration would have a real meaning." (D'Abro, 1950, p. 111)
[7] It was German scientist Leibniz who first called these intersections 'coordinates.'
[8] Another example of coordinate axes are the lines of longitude and latitude on a map measured from arbitrary zero points of reference on the Earth's surface and used to designate positions, places and events on the surface of the Earth.
[9] "In modern physics, coordinate systems are nothing but a useful fiction." (Jammer, 1954, p. 98) The abstract representation of points and figures in space by coordinates and their analysis is called 'analytic geometry.' Later, geometrical structures were described by algebraic symbols and equations (a process now called 'algebraic geometry').

dimensional distances in space by determining the magnitudes of the three spatial coordinate axes (x, y, and z) relative to the zero point, and then computing the distance from the zero point to the material object or physical event (P), again by means of the Pythagorean Theorem. [10] (Einstein, '*Relativity*,' pp. 6-9; see Figure 3.1B)

When the origin (zero point) of a coordinate system is theoretically connected to the eyes of a fictional observer, then the coordinate system can mathematically be called the 'frame of reference' for such observer.[11] (Young, p. 78) According to mathematical theory, every observer in the universe has an imaginary system of Cartesian coordinates (with a set of x, y, z axes) rigidly attached to his eyes. This fiction of a personal coordinate system theoretically enables the observer to geometrically determine the position and distance of any other body or event in space relative to his eyes. (*Id*.)

Unfortunately, the term frame of reference (or reference frame) is often used indiscriminately and interchangeably to mean several very different things. For example, it can mean a system of coordinates, the visual perspective of a human observer, a distant object that the observer or measurer is referring to for measurements, or something with a unique or specific velocity. In Special Relativity, a frame of reference is the mathematical representation of a material body of reference in <u>motion</u> with a specific velocity, which moving body can contain an infinite number of spatially separated observers or measurers, each with his own clock. All of these different meanings can become very confusing.

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[10] Thus, Descartes extended the Pythagorean Theorem to three dimensions. (Harrison, p. 134) Such three-dimensional distance measurements can be made either directly (by rigid measuring rods) or indirectly by instruments or estimations.

[11] Mathematically, bodies and moving bodies are referred to as 'frames.' A 'reference frame' is a fictional mathematical construct, which is composed of a rigid coordinate system with a hypothetical observer or measurer making space and time measurements from its zero point of origin. (Rohrlich, p. 20; see Figure 3.2)

The frame of reference of an observer may be stationary relative to the position of an object or an event, or it may involve a relative change of position—by the object, by the event, and/or by the observer. This relative change of position, especially if it is continuous, is usually referred to as 'motion.' (see Memo 3.3) When a time scale or other method for measuring an instant or interval of time (i.e. a clock) is added to the coordinate system, the motion or relative change of position of a frame over time can be mathematically described. [12]

How can we graphically illustrate the motion of a material point or body on a system of coordinates? First, we must determine the successive positions of the body at successive specified times. (Born, pp. 16 – 17) To do this, we must change the x-axis of the coordinate system to represent the distance (d) traveled by the body, and we must change its y-axis to represent an interval of time elapsed (t). (*Id*., p. 18) We then plot each successive position of the moving body as a point for each successive interval of time. By connecting the points a line is created which represents and illustrates the motion of the moving body. In this way we can abstractly illustrate various different types of motion. (*Id*., pp. 16 – 17; Figure 3.2)

Coordinate systems can be used to analyze the motions of matter in the abstract or with regard to their causes; in other words, by kinematics or dynamics. '<u>Kinematics</u>' is the study of abstract geometrical motions of bodies, without consideration of their causes (i.e. a force). '<u>Dynamics</u>' is the study of motion with regard to its causes, i.e. the magnitudes of force, mass, and time which result in the acceleration, velocity, or

[12] Thus 'motion' can mathematically be defined as the change of the position of a rigid body during an interval of time, as measured by an observer at a particular position with a particular frame of reference. (see Oxford Dictionary of Physics, p. 309) Motion can also be described as the 'translation' of a body from one position along a straight line to another position. (French, p. 67)

momentum of a body. (Folsing, p. 178)

The motions of bodies may be characterized by their magnitudes. (see Memo 3.3) The 'speed' of a body is the rate of its motion…the distance a body moves from one position to another, divided by the time elapsed. The 'velocity' of a body (or a ray of light) expresses its speed in a certain direction. [13] (Goldberg, p. 33) When the rate of speed of an object is uniform or constant and in the same rectilinear direction, this can be called a 'uniform rectilinear velocity.'[14] (see Figure 3.2B) A uniform rectilinear velocity may result from a continuous force, such as where a locomotive is needed to propel a train down a straight track at a constant speed in order to overcome friction and the Earth's force of gravity. However, in empty space, far from gravity and friction, this continuous application of force is not necessary to maintain a substantially uniform rectilinear velocity. This type of uniform rectilinear velocity of a celestial or other body in space (without the application of force) is known as 'inertial' motion. (see Figure 4.1A) Because Special Relativity is based upon 'inertial motion' (with or without the application of force), inertial motion will become very important for our later discussions.

When either the rate of the speed or the direction (orientation) of a body's uniform velocity is changed (vis., by another force), this changed motion is known as 'acceleration.'[15] Accelerations may be 'uniform,' such as a train uniformly increasing its speed down a straight track (see Figure 3.2C), or the uniform circular orbit of a body (Figure 3.4C1), or the constantly increasing gravitational acceleration of a body falling

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[13] Speed is called a 'scalar' quantity, because only the magnitude of a body's speed is important. Velocity is called a 'vector' quantity because the direction of the body is also important. (Goldberg, p. 33)
[14] Galileo was the first person to realize that "a uniform velocity is a constant rate of change of position." (Born, p. 7)
[15] Mathematicians often refer to 'deceleration' (a decrease in speed or velocity) as 'acceleration in a negative direction.' (see Figure 3.2E)

toward Earth.[16] (see Figure 4.1B) Accelerations may also be varied or arbitrary, such as the herky-jerky motions of a roller coaster. (see Figure 3.2D)

Motions of bodies may also be characterized by their orientations: their directions of motion. For example, 'rectilinear' or 'translational' motion is where a body moves or 'translates' in a straight line from one position to another.[17] (see Figure 3.4A) 'Curvilinear' motion is where a body moves from one position to another in an arc. (see Figure 3.4B) 'Orbital' motion is where one body moves around another body in a circular, elliptical or arbitrary path. (see Figure 3.4C) 'Rotational' motion is where all points on a body rotate or revolve around the body's own axis. (see Figure 3.4D) The motion of the Earth through space exhibits a combination of all of the above described motions, orientations and trajectories at the same time: relative to its own axis, and relative to other planets, the Moon, the Sun, other stars, the core of the Milky Way Galaxy, and other galaxies.

B. Trajectories, Perspectives, Perceptions, & Transformations

What is a trajectory? It is usually described as the path of a moving object, such as a baseball in flight. The trajectory of an object may be more specifically described as the continuance over space and during time of the three-dimensional coordinate positions of such object as perceived or measured by an observer.

The trajectory of any moving terrestrial body will appear to be different for each observer, depending upon such observer's unique position and visual perspective relative to such moving body. For example, a baseball slugger views the curved trajectory of his

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[16] Again, Galileo was the first person to realize that "a uniform acceleration is a constant rate of change of velocity." (Born, p. 7)

[17] The mathematical term 'translation' means: 'Motion of a body in which all the points in the body follow parallel paths." (Oxford Dictionary of Physics, p. 507)

home run ball to be in a rectilinear orientation relative to his unique position and visual perspective in the stadium. (see Figure 3.5A) When other observers choose different unique positions in the stadium, their visual perspectives, their mental perceptions and their frames of reference concerning the trajectory of the home run ball will necessarily be quite different. (see Figures 3.5B, 3.5C and 3.5D)

Does this mean that the physical laws of motion for the ball, during its flight from the batter to the bleachers, are different for each individual observer? Of course not. There is only <u>one motion</u> of the baseball, and this motion does <u>not vary</u> because of different observers watching or measuring it. Only each observer's unique visual perspective, perception and <u>coordinate description</u> of such motion (its trajectory relative to the observer's eyes and his unique position) varies. (see Rohrlich, pp. 20 – 21)

These simple concepts and facts will become critical when we consider and analyze Special Relativity in Part II of this treatise. Why? Because Einstein claimed *ad hoc* that his dubious methods of measurement from one moving body to or from another distant moving body caused the <u>length</u> of a meter rod to become shorter or contracted (possibly to zero length) on the distant moving body, and also caused the <u>duration</u> of the time intervals on such distant moving body to shorten (possibly to zero duration). (see Figure 3.8 and Chapters 26 & 28) Later Einstein even claimed that such dubious perceptions and measurements caused the magnitude of the <u>mass</u> of the distant moving body to increase (possibly to infinity).[18] (see Chapter 31)

Now let us return to the discussion of perspectives, perceptions, and trajectories. What if the observers are also moving relative to the trajectory of the moving baseball?

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[18] Einstein also claimed *ad hoc* that (based on his measurements) a light ray had one velocity with respect to a stationary object, and a greater or lesser velocity with respect to a body that was moving linearly toward or away from the light ray. (Einstein, *Relativity*, pp. 22 – 23)

Each moving observer at different changing positions relative to the moving baseball merely has different visual perspectives, different changing perceptions and different coordinate descriptions of the same event…the same motion of the baseball. (Rohrlich, p. 20) Algebraic[19] equations that mathematically relate two different coordinate descriptions of the same (or an identical) event are called 'transformation equations.' (Goldberg, p. 74) The different coordinate descriptions of the same event that we have been discussing obviously result from the different visual perspectives and perceptions of various observers or the same moving observer. On the other hand, some different coordinate descriptions that we will later discuss in Special Relativity result from Einstein's creation of distorted transformation equations and Einstein's misapplication of such distorted transformation equations to physical phenomena.[20]

As a further example of such different perspectives, perceptions and different coordinate descriptions of the same event, assume that a stone is dropped from a uniformly and rectilinearly moving railway carriage onto the straight railway embankment below. In 1916, Einstein asserted that:

> "the stone traverses a straight line relative to a system of co-ordinates rigidly attached to the carriage, but relative to a system of co-ordinates rigidly attached to the ground (embankment) it describes a parabola. With the aid of this example it is clearly seen that there is no such thing as an independently existing trajectory (lit. 'path-curve') but only a trajectory relative to a particular body of reference."[21] (Einstein, *Relativity*, p. 11; see Figure 3.7)

The falling stone in Einstein's above description is merely an example of the stone's combined inertial motion and gravitational fall in a parabolic motion (see Figure

[19] What do we mean by the words 'algebra' and 'algebraic?' See Memo 3.6.
[20] For example, Einstein misapplied the Galilean transformation equations to light and misinterpreted the results. (Chapter 19) Then he derived and applied the radical distorting Lorentz transformation equations to light in order to attempt to rectify and justify the mischief that he had created. Both of these transformation equations will become critical to our later discussions.
[21] But see the note and question at the bottom of Figure 3.7.

4.1), expressed in terms of each observer's different visual perspectives, perceptions and Descartes' coordinates. The so-called "straight line" vertical coordinate trajectory described in Einstein's above example is best referred to as an illusion. [22] As rationalized by Born, the "stone falls…along a vertical [path] that is moving with the [carriage]. (Born, p. 69)

Why did Einstein describe this example of two different observers with different visual perspectives who perceive different trajectories for the same motion? It certainly could not have been for the reason that he suggested, because Einstein's Special Theory has nothing to do with different perceived trajectories for the same motion.[23]

Einstein's real reasons for describing the above example so early in his book *Relativity* were obviously quite different. His example attempts to establish certain mindsets for his readers with respect to his later Special Relativity concepts, analogies and rationalizations.[24] First of all, Einstein is suggesting that the relative motions (of the passenger on the train, the falling stone and the observer on the embankment) by themselves can affect each different observer's perceptions, mathematical descriptions and coordinate measurements of the same event.[25] Secondly, for his Special Theory,

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[22] From the perpendicular perspective of the eye of observer No. 1 standing on the embankment, the combined (inertial and gravitational) parabolic motion of the stone was obvious. (see Figure 3.7) But the observer No. 2 on the carriage perceives a different trajectory. From the vertical perspective of the eye of observer No. 2 on the uniformly moving carriage, the stone moves uniformly forward substantially at the same velocity as such observer's eye (because of the horizontal inertia of the stone) as it also falls downward toward the embankment. So such observer's moving eye watching the moving stone fall always sees the stone directly below his co-moving eye as if the stone was falling straight down. The resulting "straight line" vertical coordinate trajectory is a visual illusion. Einstein inferred that such different trajectories were solely the result of relative motion, which of course is nothing more than a continuous relative change of position.

[23] Instead, Special Relativity has to do with two different measurements of lengths and time intervals for one motion of an object. (see Chapters 26 and 28)

[24] It should be realized at this early juncture that Einstein seldom made any statement concerning physics unless it furthered his agenda for one of his theories.

[25] But if a slow motion movie was taken of the events from either frame of reference, then everyone would realize exactly what was happening.

Einstein needed two different observers (measurers), each with a different uniform rectilinear relative motion, and each with a different frame of reference (coordinate system).[26] His above example provided all of these requirements.

But above all, what Einstein specifically suggested with his above example was that different observers on different inertial bodies (i.e. observer No. 2 on the train and observer No. 1 on the Earth) with different inertial velocities can make different coordinate measurements of the same motion of an object entirely by reason of their relative velocity. (see Figure 18.1) In effect, Einstein's above example was a preview of Special Relativity; an early artificial analogy that attempted to indoctrinate his readers with misleading ideas that might confuse them into believing that his later meaningless relativistic concepts, i.e. the Relativity of Length, were at least plausible.

In Einstein's relativistic concept (called the Relativity of Length), the engineer in the detached engine (which is moving at a different uniform velocity than the carriage) theoretically cannot simultaneously plot the time coordinates for the coordinate positions of the front of the carriage and of the rear of the carriage by the hand and eye method. (see Figure 3.7; Resnick, 1992, pp. 480 – 481) Therefore, the engineer must plot each time and position coordinate separately. According to Einstein, another reason for this physical inability is that the light from the rear of the carriage takes a longer time interval to reach the engineer's eye than the light from the front of the carriage.

Theoretically, these physical inabilities will cause mathematical measurement problems for Einstein. During the time interval between each plotting, the carriage and the engine will have changed their relative positions (because of their different velocities)

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[26] Throughout his theories, Einstein equates the concepts of reference frames and coordinate systems. For Einstein, they are identical.

so that the plotting of the second coordinate position will be different than if the engineer and the carriage were relatively at rest. Thus, claimed Einstein, the physical length of the carriage is entirely relative for the engineer and it depends upon the relative velocity of the two moving bodies (in other words, their reference frames) and the time interval between such coordinate measurements.[27] (also see Figure 18.1)

Stated in a different way, the coordinate descriptions for the length of the carriage will be different for the engineer's delayed visual coordinate estimates of length than for the observer on the carriage who physically measures its length with a rigid meter rod. Therefore, there are two different coordinate descriptions and measurements for the length of the same moving object. Einstein's artificial 'solution' for this concocted mathematical problem of measurement was to relate such different coordinate measurements for the same event with his radical Lorentz transformation equations.[28] The Lorentz transformation equations, in turn: 1) distorted Einstein's inexact distant moving measurements so that they became mathematically identical to the measurements of the observer on the carriage, and 2) algebraically produced all of the bizarre mathematical consequences of Special Relativity, including the absolute velocity of light at *c*, length contraction, time dilation, increase in mass with velocity, etc., which Einstein needed for his Special Theory. "Technically, the whole of the special theory is contained in the Lorentz transformations." (Bertrand Russell, 1927, p. 29)

It may be difficult for the reader to believe at this point in our story, but these

[27] In short, the engineer (in his moving coordinate system) cannot physically, by Einstein's sequential coordinate method of measurement, measure the correct (stationary) physical length of the moving carriage in its differently moving coordinate system.

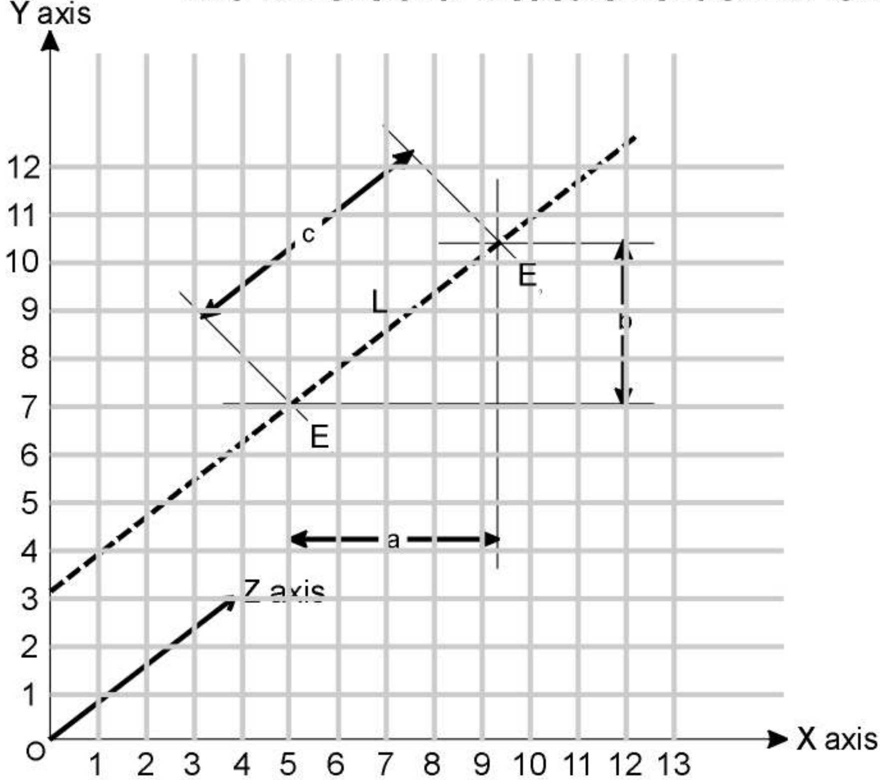
[28] The Lorentz transformation equations were invented by Dutch physicist H. A. Lorentz during the period 1899 – 1904 in an *ad hoc* attempt to mathematically rationalize the Michelson & Morley paradox and other paradoxes caused by the ether theory. We will discuss these subjects in detail in Chapters 9 – 12, 15 and 16.

theoretical and technical inabilities to make simultaneous mathematical coordinate measurements for both ends of a moving object in 1905 were the basic mathematical premise and rationalization for all of Einstein's contrived relativistic theories of measurement.[29] (see Chapter 28) They were also the primary mathematical justification for his entire Special Theory.[30]

Why are we even discussing the ancient concepts of coordinates, frames of reference, transformation equations, relativity, and the like in this chapter? Because they are all man-made conventions that were invented during centuries past in order to physically and mathematically describe and understand the positions, motions, relationships and measurements of material bodies in space and during time.[31] Antiquated as these mathematical conventions may be, they are all necessary for a basic understanding of Einstein's Special Theory of Relativity as well as other theories and mathematical concepts that we will discuss in later chapters.

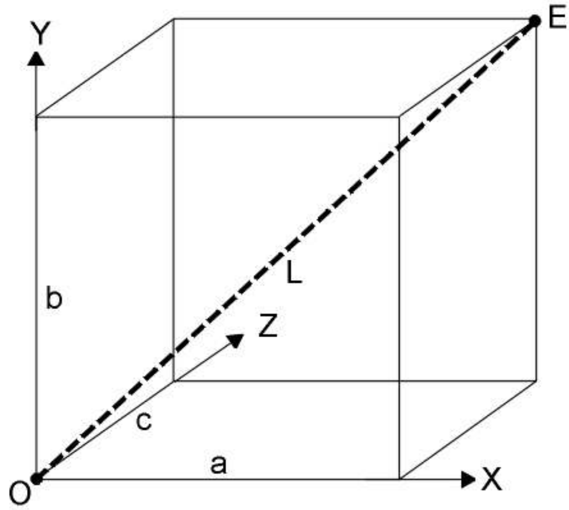
[29] In addition, Einstein asserted that such inability to simultaneously measure the instant of time for such coordinate positions also meant that time was entirely relative for the engineer. This concocted reciprocal concept was called the Relativity of Simultaneity, or the Relativity of Time. (see Chapters 26 and 28)
[30] One might ask: What do Einstein's physical inabilities to make exact measurements by hand and eye coordinate plottings in 1905 have to do with physics in the 21st century? The answer is: nothing.
[31] It should be emphasized that the aforementioned concepts of coordinates, frames of reference, transformations and relativity are merely abstract mathematical conventions invented for purposes of mathematical description and theoretical measurement. Whereas, the concepts of positions, a body of reference, rigid measuring rods, trajectories, changes of position over time (motions), distances (lengths) and perspectives are all observed, empirical and physically real.

Figure 3.1A Descartes' System Of Coordinates Showing A Two-Dimensional Measurement On A Plane



To determine the position of a body or event (E_2) on a two-dimensional plane (i.e. the surface of the Earth), the distance (a) is measured along the X axis (length). The distance (b) is then measured along the Y axis (width). The distance or length (L) between event E_1 (a point, event or body of reference on the Earth) and event E_2 (the body or event in question) is measured and computed by applying the Pythagorean theorem: $a^2 + b^2 = L^2$. The Z axis is only used for three dimensions.

Figure 3.1B Three-Dimensional Measurement in Space



To determine the position of an object or event (E) in three-dimensional space, the measurement of distance (c) along the Y axis (height) must also be made. Then the distance or length (L) from O to E can be determined by applying the Pythagorean theorem: $a^2 + b^2 + c^2 = L^2$.

Figure 3.1 Cartesian Coordinates Along With The Pythagorean Theorem Geometrically Measures Distances

Sources: Harrison, p. 126; Einstein, Relativity, pp. 6-8; Born, p. 8

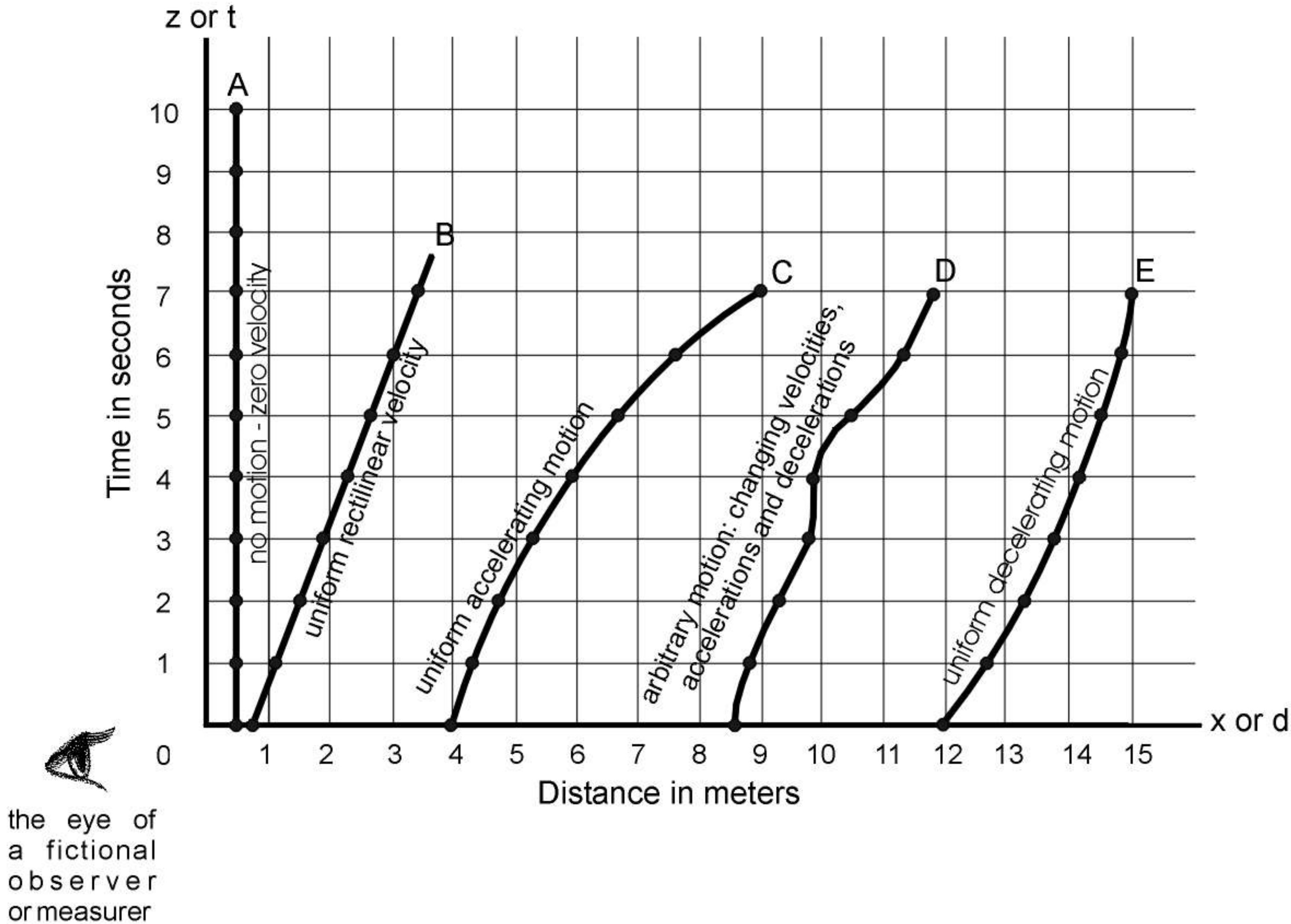


Figure 3.2 Abstract Motions Of Bodies Depicted On The Cartesian Coordinates Of A Frame Of Reference (in meters/second)

MEMO 3.3

What is the difference between the following concepts: motion, speed, velocity, acceleration, inertial motion, relative motion, relative velocity, displacement, and relative displacement?

> Motion is "the passage of a body from one place to another." (Webster's Dictionary, p. 886)
>
> Speed is "the rate of movement or motion." (*Id*., p. 1288) Specifically, 'speed' is the distance that a body moves from one position to another position, divided by the time elapsed: S = d/t.
>
> Velocity is "the rate of change of position [of a body] in relation to time…in a particular direction." (*Id*., p. 1479)
>
> Uniform rectilinear velocity is where the speed of a body is constant in the same rectilinear direction.
>
> Acceleration occurs when the speed or the direction of a body's velocity is changed.
>
> Inertial motion (in classical physics) is where the uniform rectilinear velocity of a body does not result from an apparent force.[1]
>
> Relative motion is the motion of one body relative to the motion of another body.
>
> Relative velocity is the rate of change of position of one body (or thing) in a particular direction as compared to the rate of change of position of another body (or thing) in its particular direction.
>
> Displacement is "the difference [in distance] between a later position of a thing and its original position." (*Id*., p. 396)
>
> Relative displacement is the distance of displacement of one body or thing relative to another body or thing.

[1] In Special Relativity inertial motion may result from a force.

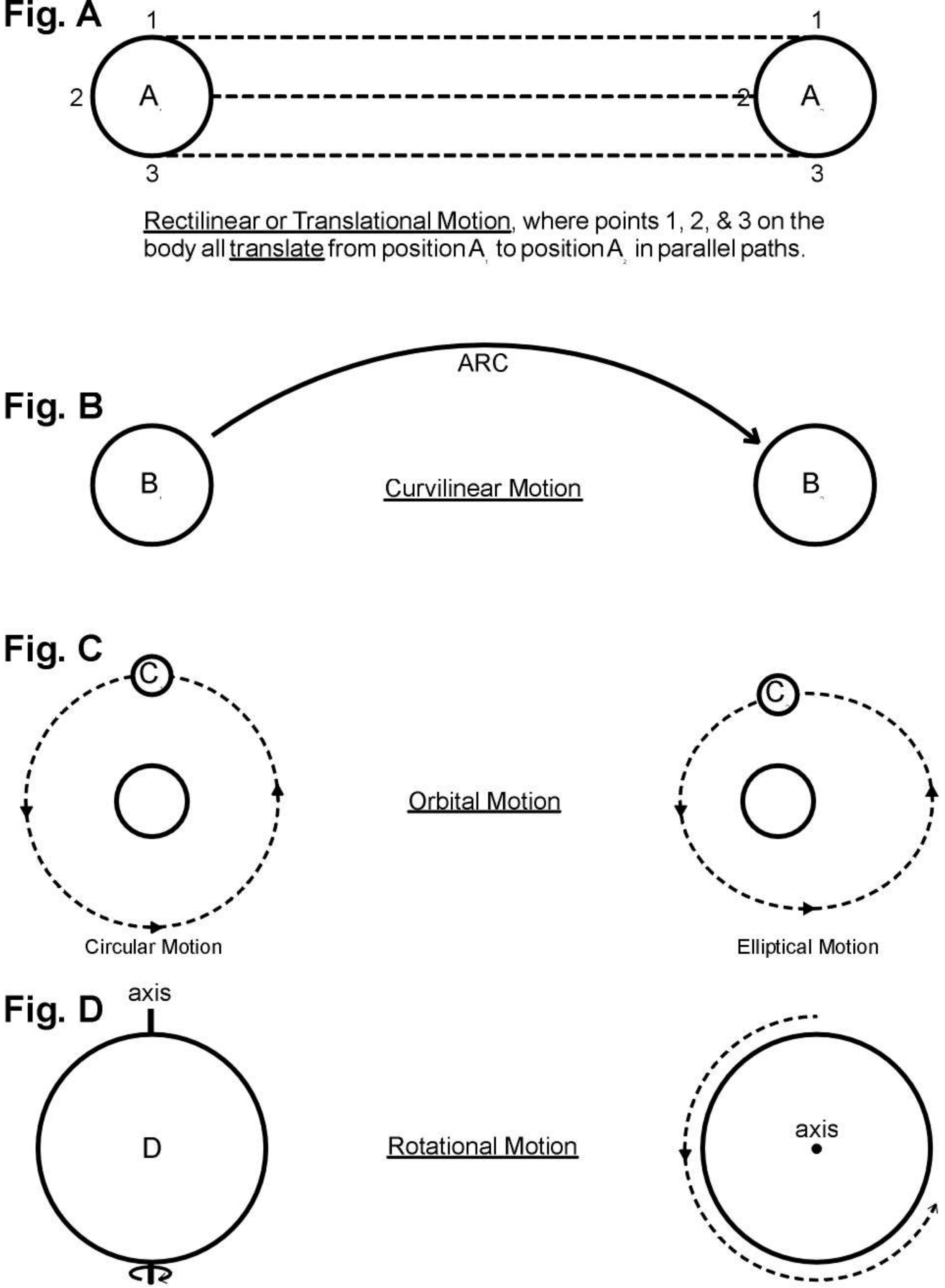


Figure 3.4 Motions Of Bodies According To Their Orientation

Figure 3.5 Different Observed Trajectories For The Same Motion

How is the motion (trajectory) of a home run baseball perceived by different observers at different positions (locations) with different perspectives and frames of reference?

Fig. A From the position of the batter

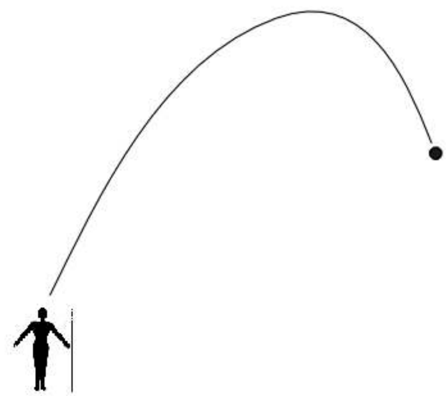


Fig. B From the position of the spectator to the far right of the batter

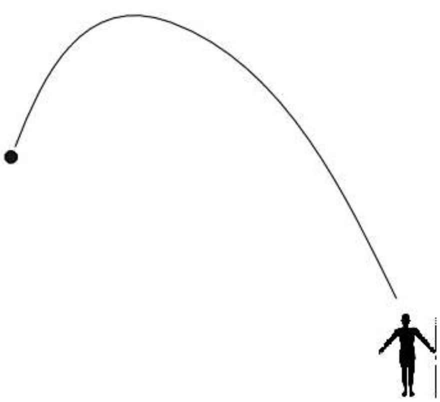


Fig. C From the position of the spectator to the far left of the batter

Fig. D From the position of the man in the blimp almost directly overhead

MEMO 3.6

What is meant by the words ‘algebra’ and ‘algebraic’? To the layman or non-mathematician, they mean that branch of mathematics which originated in Babylonia (around 1600 BCE) and in which one learns to calculate an unknown quantity with abstract variables and negative values instead of just the positive numbers of arithmetic. (E.B., Vol. 1, p. 607) The word ‘algebraic’ may also mean the substitution of symbols or letters for concepts in order to understand or express by mathematical formulas or equations more clearly and simply the relationships between such concepts.

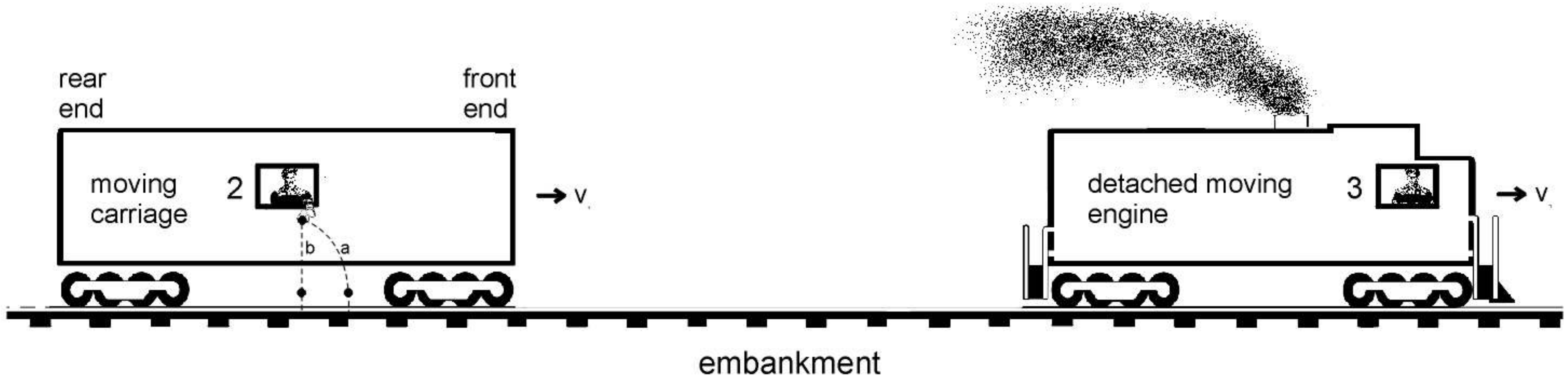
On the other hand, with respect to the 19th and 20th century pure mathematician or scientist, the word ‘algebraic’ also means and involves the sophisticated theoretical study of abstract mathematical structures (such as fields, lattices, and vector spaces, etc.) that are constructed with complicated mathematical axioms (i.e. formal rules of operation and defined relationships) often in multiple theoretical dimensions. (see *Id.*, pp. 607 – 608) This highly theoretical, abstract and axiomatic point of view continues to this day.

In any event, it is essential with all forms of algebra “that the calculations…involve only a finite number of quantities and end after a finite number of steps.” (*Id.*, p. 607) Therefore, the concepts of ‘infinity’ and ‘eternity’ have no place in algebra.

Figure 3.7 The Trajectory Of A Stone Falling From A Moving Train, As Perceived By The Eyes Of Different Spatially Separated Observers Located At Different Positions

a. Relatively stationary observer 1 on the embankment perceives the trajectory of the stone as a parabola (a).
b. The man (observer 2) dropping the stone from the carriage moving at v_1 perceives its trajectory as vertical (b).
c. The engineer (observer 3) in the detached engine moving at v_2 may perceive a somewhat different trajectory of the stone than observer 2.

Why? Because each observer has a different position, a different visual perspective and perception, and a different relative motion; in other words, a different frame of reference.



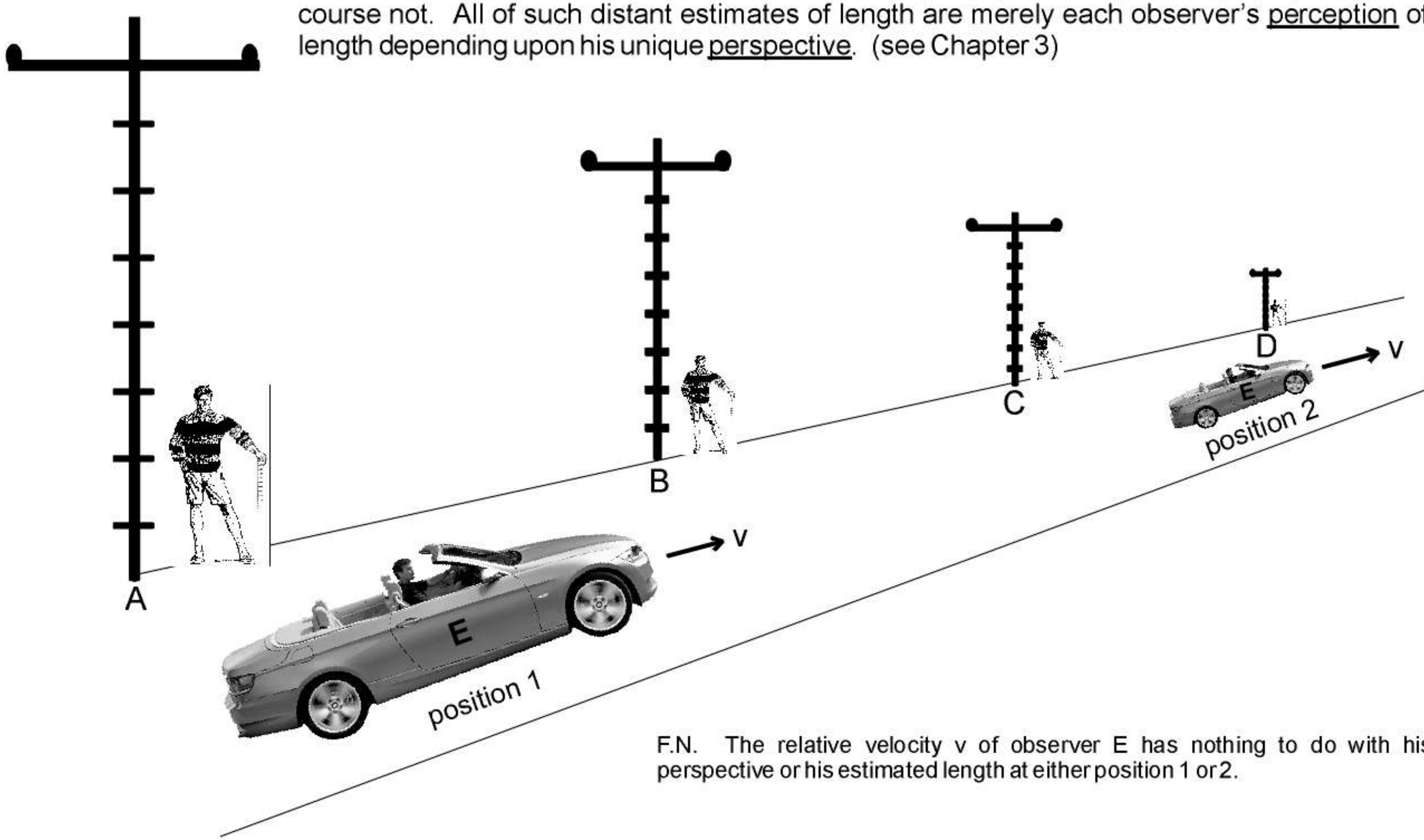
All of the above spacially separated observers also perceive a slightly different instant in time that the stone leaves the man's hand (at 2), because of the slightly different <u>distance</u>/time interval delay of light from the stone to each observer's eyes.

[Note: Is a body of reference really necessary for the stone's trajectory? If the stone was a luminous object and was dropped in the pitch black of night, all of such observers would still perceive the same trajectories of the luminous stone even though they could not observe the moving train or the embankment. In this situation, there would be <u>no body of reference</u> other than the stone.]

Partial Source: Einstein, *Relativity*, p. 11

Each observer (A, B, C & D) standing next to a telephone pole would measure its length with his meter rod to be 10 meters long. Observer A properly measures the length of pole A to be 10 meters long with his meter rod. But distant observer B with a different perspective would estimate that pole A was only 6 meters long, distant observer C with a different perspective would estimate that pole A was only 3 meters long, and distant observer D with a different perspective would estimate that pole A was only 1.5 meters long. Likewise, a driver in car E with a different persepctive would estimate the length of pole A to be greater at position 1 than later at distant position 2.[F.N.]

Are all of these estimated measurements of the length of pole A the <u>real length</u> of pole A? Of course not. All of such distant estimates of length are merely each observer's <u>perception</u> of length depending upon his unique <u>perspective</u>. (see Chapter 3)



F.N. The relative velocity v of observer E has nothing to do with his perspective or his estimated length at either position 1 or 2.

Figure 3.8 Different Perspectives And Measured Lengths Of The Same Pole A By Various Distant Observers